Qubits Europe 2019

Flight Gate Assignment with a Quantum Annealer

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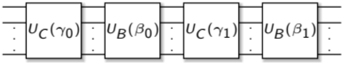
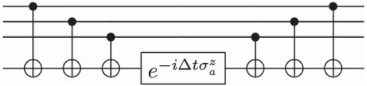
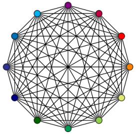
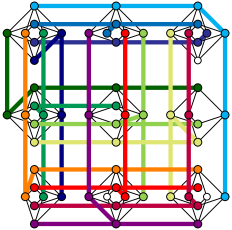
Algorithmic Quantum Computing Research at DLR

- Quantum Optimization Algorithms
- Quantum Compiling
- Embedding strategies for Quantum Annealing
  - Complete graph in broken Chimera
  - Weight distribution problem









Aerospace Applications at DLR

for Quantum Annealing

- Air Traffic Management
- Satellite Telemetry Verification
- Earth Observation Mission Planning
- Flight Gate Assignment

for Gate-Based Quantum Computing

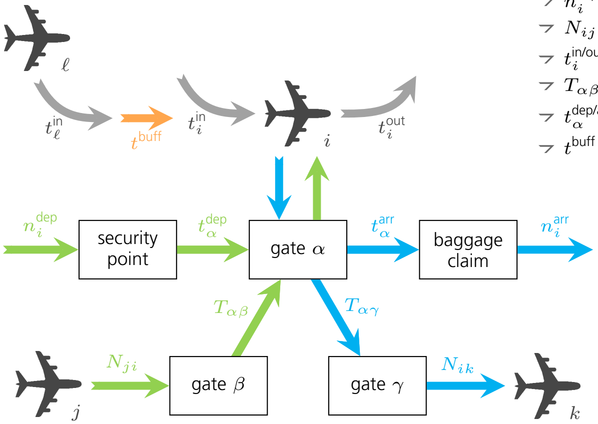
- QAOA for scheduling problems
- HHL for Radar Cross Section
- Quantum Simulation for Battery Research

Flight Gate Assignment

A day at Frankfurt Airport

- about 1300 aircraft movements (arrival and departure)
- more than 90% are passenger flights
- more than 170000 passengers
- about 60% transfer passengers
- 278 gates

Passenger Flows



- F, G sets of flights and gates
- $n_i^{\text{dep/arr}}$ passengers which depart/ arrive with flight i
- N_{ij} transfer passengers from flight i to j
- $t_i^{\text{in/out}}$ arrival/departure time of flight i
- $T_{\alpha\beta}$ average time to get from gate α to β
- $t_\alpha^{\text{dep/arr}}$ average time to arrive at/ leave from gate α
- t^{buff} buffer time between two flights at the same gate

Which flight should be assigned to which gate, such that the total transit time of the passengers is minimal?

$$A : F \to G$$

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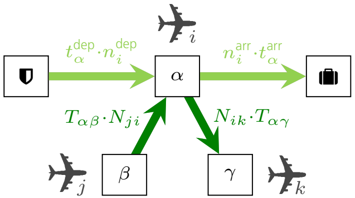
FGA Binary Program



Variables $x \in \{0,1\}^{F \times G}$ with

$$x_{i\alpha} = \begin{cases} 1, & \text{if flight } i \text{ takes gate } \alpha, \\ 0, & \text{otherwise} \end{cases}$$

Minimizing the total transfer time with objective function

$$T(x) = T_{\text{arr}}(x) + T_{\text{dep}}(x) + T_{\text{transfer}}(x)$$

$$= \underbrace{\sum_{i\alpha} n_i^{\text{arr}} t_\alpha^{\text{arr}}\, x_{i\alpha} + \sum_{i\alpha} n_i^{\text{dep}} t_\alpha^{\text{dep}}\, x_{i\alpha}}_{\text{linear}} + \underbrace{\sum_{ij\alpha\beta} N_{ij} T_{\alpha\beta}\, x_{i\alpha}\, x_{j\beta}}_{\text{quadratic}}$$

$\Rightarrow$ Quadratic Assignment Problem

- fundamental problem in combinatorial optimization, NP-hard
- seems to exploit possible advantages of the D-Wave machine

Constraints and Penalty Terms

1. One gate per flight $$\sum_{\alpha} x_{i\alpha} = 1 \qquad \forall i \in F$$

2. Different gates if standing times of two flights overlap forbidden pairs

$$P = \left\{ (i,j) \in F^2 : t_i^{\text{in}} < t_j^{\text{in}} < t_i^{\text{out}} + t^{\text{buff}} \right\}$$

$$x_{i\alpha} + x_{j\alpha} \leq 1 \;\Leftrightarrow\; x_{i\alpha} \cdot x_{j\alpha} = 0 \qquad \forall (i,j) \in P \;\forall \alpha \in G$$

$\Rightarrow$ Penalty terms $C_{\text{one}}(x) = \sum_{i} \left(\sum_{\alpha} x_{i\alpha} - 1 \right)^2,$

$$C_{\text{not}}(x) = \sum_{\alpha} \sum_{(i,j) \in P} x_{i\alpha} x_{j\alpha} \quad \text{where} \quad C_{\text{one/not}} \begin{cases} > 0, & \text{if constraint is violated} \\ = 0, & \text{if constraint is fulfilled} \end{cases}$$

QUBO with Penalty Weights

$$Q(x) = T(x) + \lambda_{\text{one}} C_{\text{one}}(x) + \lambda_{\text{not}} C_{\text{not}}(x)$$

Need to ensure that a solution always fulfills constraints, hence $\Delta C > \Delta T$

⇒ Comparing coefficients in worst cases for

- not assigning a flight to any gate

$$\lambda_{\text{one}} > \max_{i,\alpha} \left(n_i^{\text{dep}} t_\alpha^{\text{dep}} + n_i^{\text{arr}} t_\alpha^{\text{arr}} + \max_\beta T_{\alpha\beta} \sum_j N_{ij} \right)$$

- assigning a pair of forbidden flights to the same gate

$$\lambda_{\text{not}} > \max_{i,\alpha,\gamma} \left(n_i^{\text{dep}} t_\alpha^{\text{dep}} - n_i^{\text{dep}} t_\gamma^{\text{dep}} + n_i^{\text{arr}} t_\alpha^{\text{arr}} - n_i^{\text{arr}} t_\gamma^{\text{arr}} + \max_\beta \left(T_{\alpha\beta} - T_{\gamma\beta} \right) \sum_j N_{ij} \right)$$

⇒ Refinement by bisection of weights yielding valid or invalid solutions

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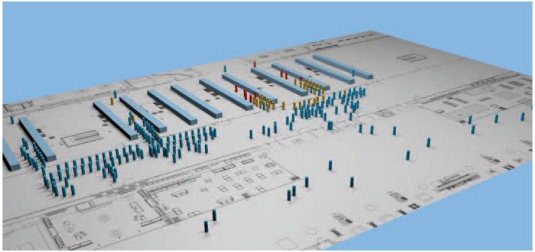
Airport Data

- Flight schedule for one day from a mid-sized European airport
- Passenger flow from agent-based simulation of Martin Jung
- Extracted total instance: 293 flights and 97 gates

⇒ Over 28000 binary variables with about 400 Mio. couplings



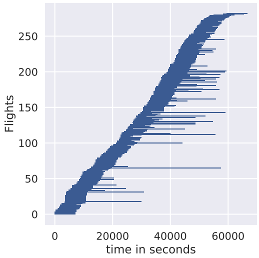
M. Jung et al. (DLR-FW)

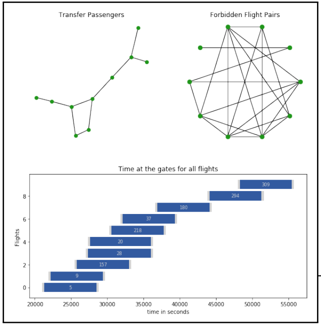
Instance Preprocessing

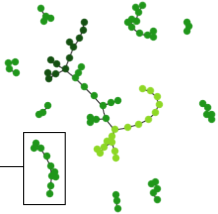
- Splitting too long on-block times
- Reducing to only flights with transfers
- Extracting connected subgraphs
- Further slicing of largest subgraph randomly

⇒ 163 instances:

- 3 to 16 flights
- 2 to 16 gates

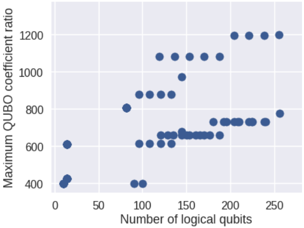






flights with transfer passengers

Bin Packing



- Maximum coefficient ratio of QUBO $C_Q = \dfrac{\max_{ij} |Q_{ij}|}{\min_{ij} |Q_{ij}|}$

- Reducing maximum coefficient ratio to overcome precision issues

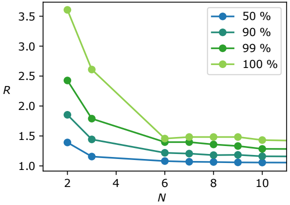
$$T_{\alpha\beta},\, t_\alpha^{\text{arr}},\, t_\alpha^{\text{dep}} \;\rightarrow\; \{0, 1, \ldots, \boldsymbol{T}\} \text{ with } \boldsymbol{T} \in \{2, 3, 6, 10\}$$

$$N_{ij},\, n_i^{\text{arr}},\, n_i^{\text{dep}} \;\rightarrow\; \{0, 1, \ldots, \boldsymbol{N}\} \text{ with } \boldsymbol{N} \in \{2, 3, 6, 10\}$$

$$\Rightarrow C_{\bar{Q}} \ll C_Q$$

- Approximation ratio (solved with SCIP)

$$R = \frac{Q(\arg\min_x \bar{Q}(x))}{\min_x Q(x)}$$



$\Rightarrow$ Little effect on solution quality

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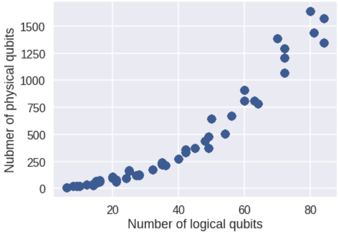
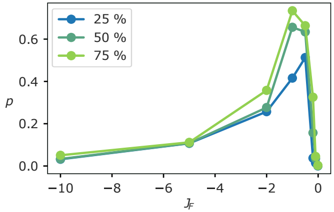
Annealing Setup

- Embedding
  - Quadratic overhead
  - Up to 84 logical qubits
    $(\#Variables = \#Flights \cdot \#Gates)$

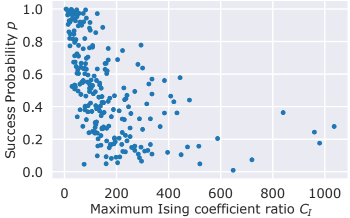


- Intra-logical coupling (J_F)
  - Influences success probability p
  - Best option by scanning: -1 in units of largest coefficient

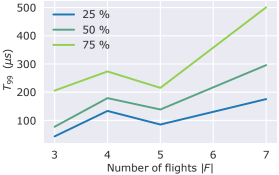


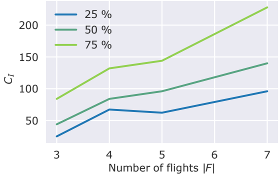
- (Standard) Run parameters
  - Annealing time $20\mu s$ with 1000 runs
  - Majority voting

Annealing Results



- QUBO to Ising transformation
  - increases maximum coefficient ratio significantly
  - ⇒ large Ising coefficients suppress success probability
- Time to solution with 99% certainty $T_{99} = \log_{1-p}(1-0.99)T_{\text{anneal}}$
  - grows with problem size → because of larger coefficients?
  - due to small problem sizes asymptotic behaviour unclear





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Summary

- Flight gate assignment is amenable to QA
- Precision issues due to large coefficients
- Mitigate limited precision by bin packing
- Open questions:
  - How to recombine partial solutions?
  - How would larger instance perform?
  - Are these instances hard for classical solvers?

Questions?

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Related Article:

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T. Stollenwerk, E. Lobe and M. Jung, QTOP, Springer, 2019

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