

# Leveraging Quantum Annealing for Large MIMO Processing in Centralized Radio Access Networks

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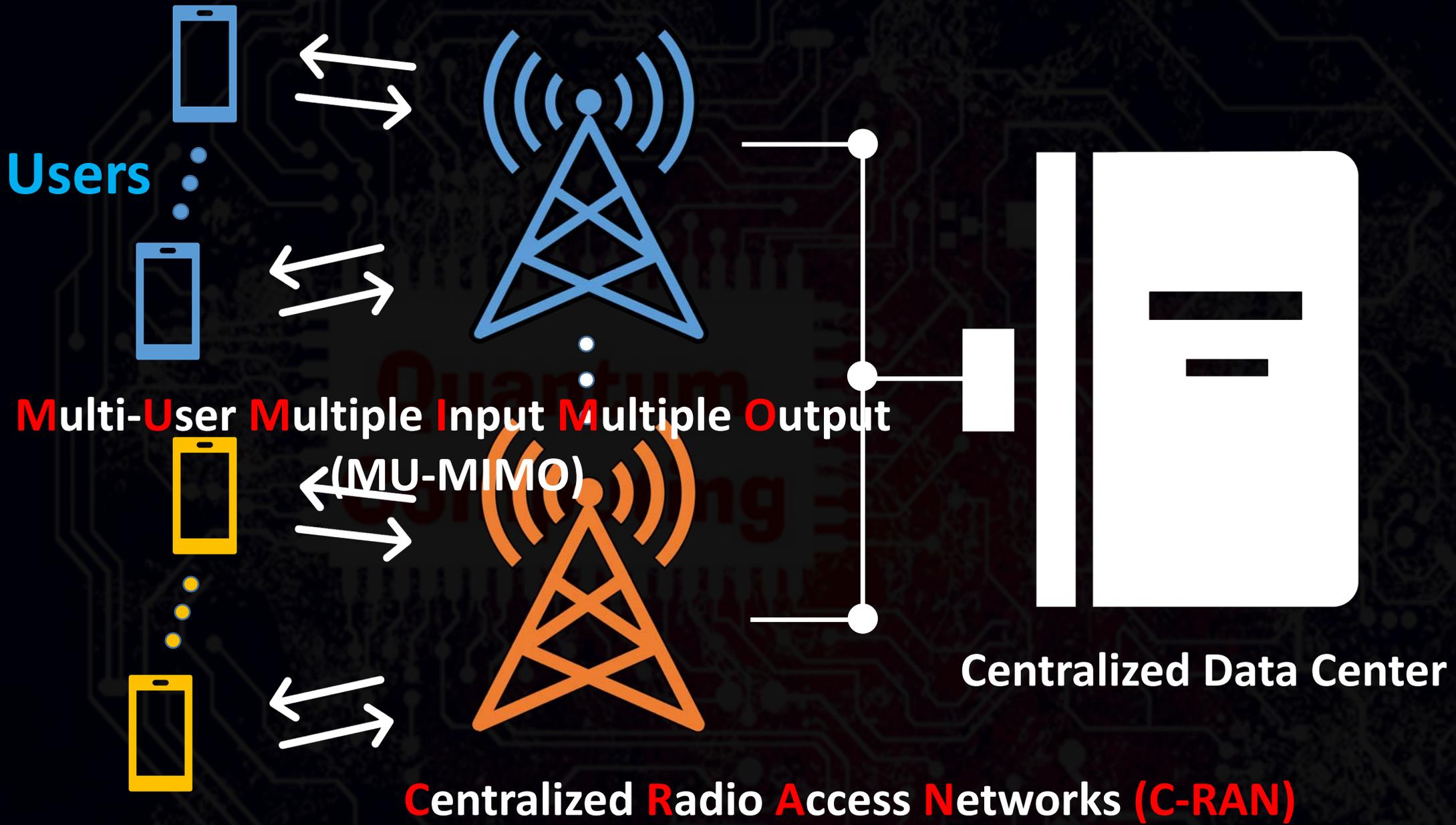
Minsung Kim, Davide Venturelli, Kyle Jamieson



# New Services!

- Global mobile data traffic is increasing **exponentially**
- User demand for high data rate outpaces supply

**Drives Wireless Capacity to increase!**



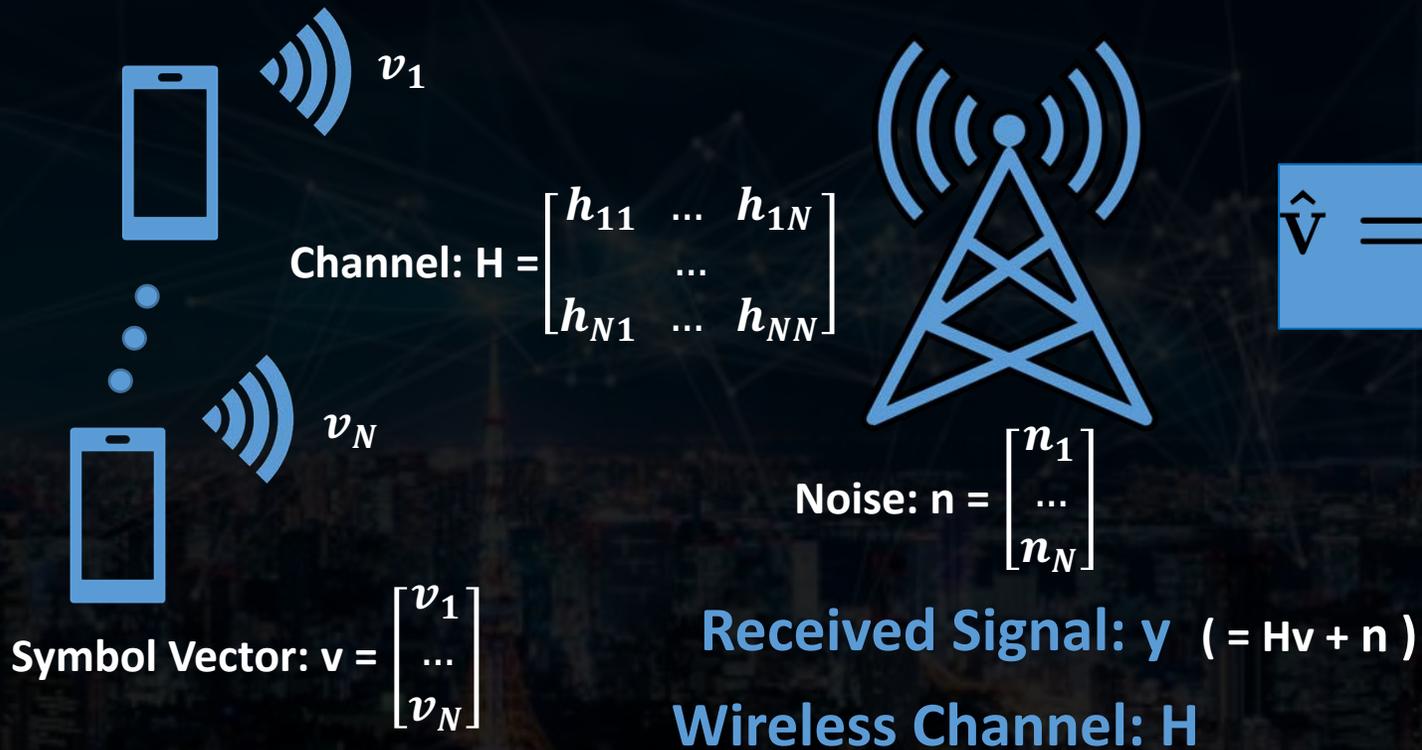
# MIMO Detection



**Demultiplex Mutually Interfering Streams**

# Maximum Likelihood (ML) MIMO Detection

: **Non-Approximate** but **High Complexity**



$$\hat{\mathbf{v}} = \arg \min_{\text{possible } \mathbf{v}} \|\mathbf{y} - \mathbf{H}\mathbf{v}\|^2$$

$2^{N \log_2 M}$  possibilities for  
 $N \times N$  MIMO with  $M$  modulation

**Time available for processing is at most 3-10 ms.**

# Sphere Decoder (SD)

: **Non-Approximate** but **High Complexity**

Maximum Likelihood (ML) Detection



Tree Search with Constraints

**Reduce search operations but fall short for the same reason**

BPSK	QPSK	16-QAM	Complexity (Visited Nodes)
$12 \times 12$	$7 \times 7$	$4 \times 4$	$\approx 40$ (♥)
$21 \times 21$	$11 \times 11$	$6 \times 6$	$\approx 270$ (Δ)
$30 \times 30$	$15 \times 15$	$8 \times 8$	$\approx 1900$ (×)

## Parallelization of SD

[Flexcore, NSDI 17],  
[Geosphere, SIGCOMM 14],

...

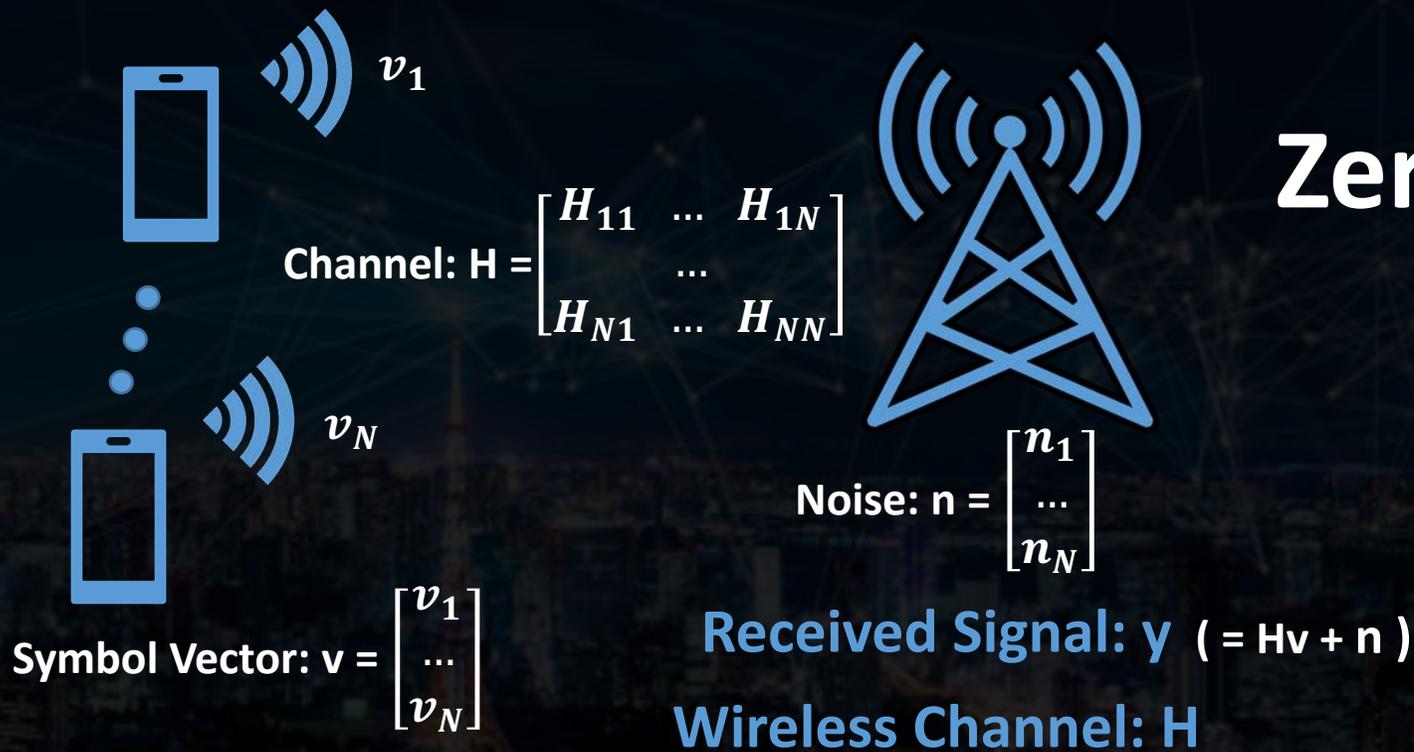
## Approximate SD

[K-best SD, JSAC 06],  
[Fixed Complexity SD, TWC 08],

....

# Linear Detection

: **Low Complexity** but **Approximate & Suboptimal**



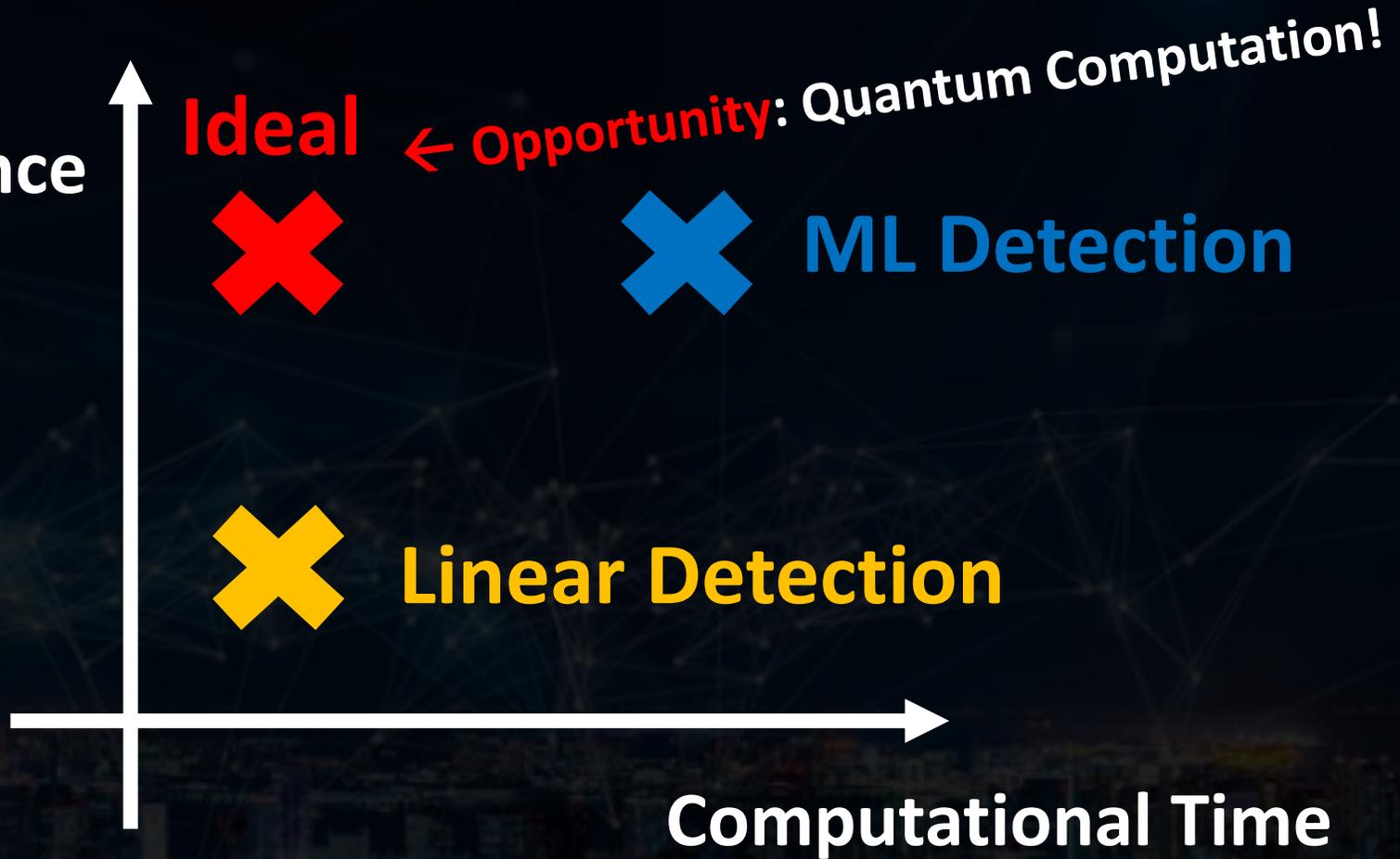
**Zero-Forcing** [BigStation, SIGCOMM 13],  
[Argos, MOBICOM 12],  
...

Nullifying Channel Effect:

$$\mathbf{H}^{-1}\mathbf{y} = \mathbf{H}^{-1}\mathbf{H}\mathbf{v} + \mathbf{H}^{-1}\mathbf{n}$$

**Performance Degradation due to Noise Amplification**

**Performance**  
high throughput  
low bit error rate



**Ideal**

← **Opportunity: Quantum Computation!**



**ML Detection**



**Linear Detection**

**Computational Time**

**Ideal: High Performance & Low Computational Time**

# QuAMax: Main Idea

**MIMO Detection**

Maximum Likelihood (ML) Detection

**Quantum Computation**

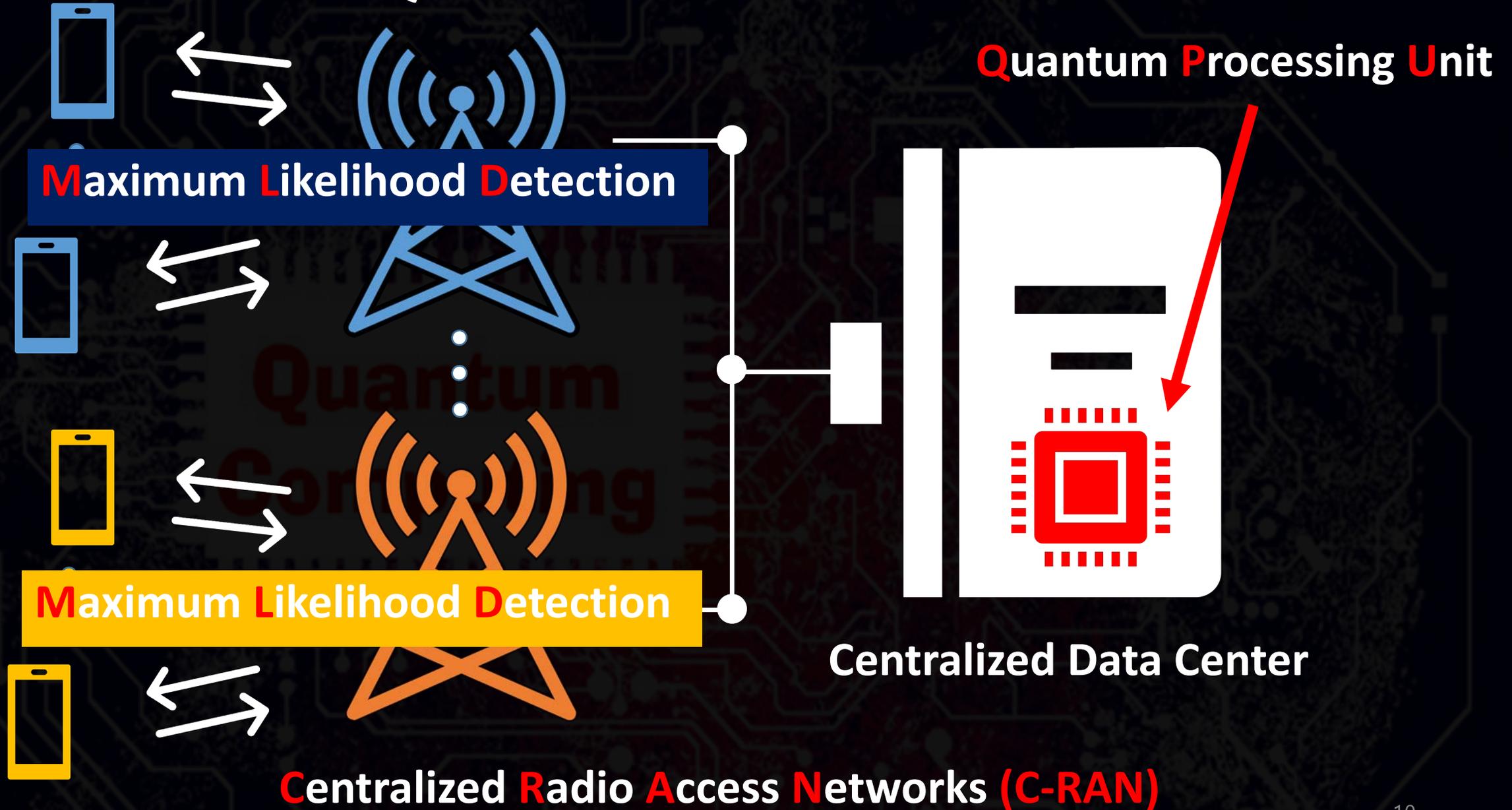
Quantum Annealing



**Better Performance ?**

Motivation: **Optimal + Fast Detection = Higher Capacity**

# QuAMax Architecture



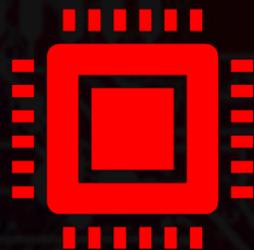
**Maximum Likelihood Detection**



**Quadratic Unconstrained Binary Optimization**



**Quantum Processing Unit**



**D-Wave 2000Q  
(Quantum Annealer)**

**1. QUAMAX: SYSTEM DESIGN**

**2. QUANTUM ANNEALING & EVALUATION**

# Key Idea of ML-to-QUBO Problem Reduction

- Maximum Likelihood MIMO detection:

$$\hat{\mathbf{v}} = \arg \min_{\mathbf{v}} \|\mathbf{y} - \mathbf{H}\mathbf{v}\|^2$$

- QUBO Form:

$$\hat{q}_1, \dots, \hat{q}_N = \arg \min_{\{q_1, \dots, q_N\}} \sum_{i \leq j}^N Q_{ij} q_i q_j$$

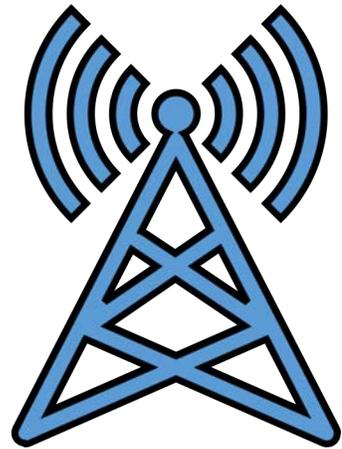
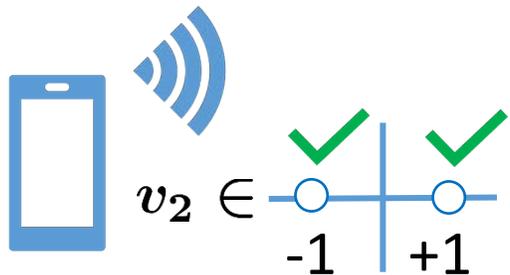
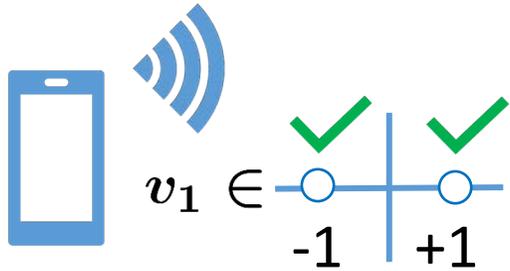
**QUBO Form!**

The key idea is to represent possibly-transmitted **symbol**  $\mathbf{v}$  with **0,1 variables**.  
If this is **linear**, the expansion of the norm results in **linear & quadratic** terms.

Linear **variable-to-symbol** transform  $T$

# Revisiting ML Detection

## Example: 2x2 MIMO with Binary Modulation



Received Signal:  $\mathbf{y}$   
Wireless Channel:  $\mathbf{H}$

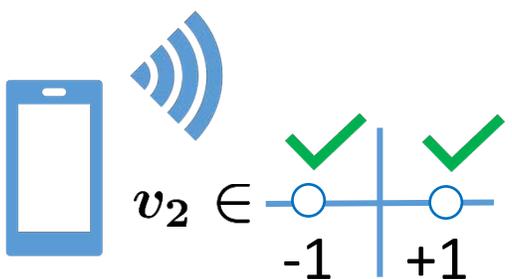
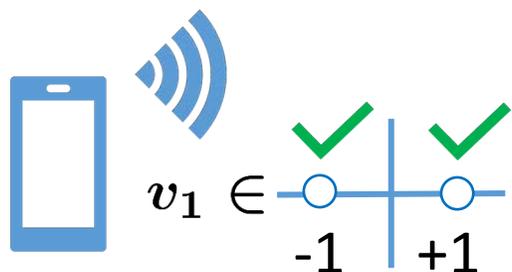
$$\hat{\mathbf{v}} = \arg \min_{\text{possible } \mathbf{v}} \|\mathbf{y} - \mathbf{H}\mathbf{v}\|^2$$

$$\text{possible } \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \in \begin{bmatrix} +1 \\ +1 \end{bmatrix}, \begin{bmatrix} +1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ +1 \end{bmatrix}$$

Symbol Vector:  $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$

# QuAMax's ML-to-QUBO Problem Reduction

## Example: 2x2 MIMO with Binary Modulation



1. Find linear **variable-to-symbol** transform T:

$$2q_i - 1 \leftrightarrow v_i \quad \begin{array}{l} \text{(if } q_i = 1) \quad 2q_i - 1 = +1 \\ \text{(if } q_i = 0) \quad 2q_i - 1 = -1 \end{array}$$

2. Replace symbol vector  $\mathbf{v}$  with transform T in  $\|\mathbf{y} - \mathbf{H}\mathbf{v}\|^2$ :

$$\text{possible } \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \in \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \iff \text{possible } \begin{bmatrix} 2q_1 - 1 \\ 2q_2 - 1 \end{bmatrix} \in \begin{bmatrix} +1 \\ +1 \end{bmatrix}, \begin{bmatrix} +1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ +1 \end{bmatrix}$$

3. Expand the norm ( $q^2 = q$ )

$$\hat{q}_1, \hat{q}_2 = \arg \min_{q_1, q_2} f_1(\mathbf{H}, \mathbf{y})q_1 + f_2(\mathbf{H}, \mathbf{y})q_2 + g_{12}(\mathbf{H})q_1q_2$$

Symbol Vector:  $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$

$$Q = \begin{bmatrix} f_1(\mathbf{H}, \mathbf{y}) & g_{12}(\mathbf{H}) \\ 0 & f_2(\mathbf{H}, \mathbf{y}) \end{bmatrix}$$

**QUBO Form!**

## QuAMax's linear **variable-to-symbol** Transform T

BPSK (2 symbols)  $v_i \leftrightarrow 2q_i - 1$

QPSK (4 symbols)  $v_i \leftrightarrow 2q_{2i-1} - 1 + j(2q_{2i} - 1)$

16-QAM (16 symbols)  $v_i \leftrightarrow 3q_{4i-3} - 2q_{4i-2} - 1 + j(3q_{4i-1} - 2q_{4i} - 1)$

- Coefficient functions  $f(H, y)$  and  $g(H)$  are generalized for different modulations.
- Computation required for ML-to-QUBO reduction is insignificant.

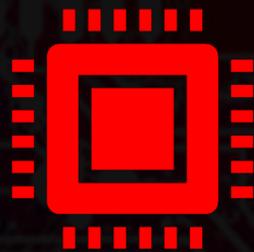
**Maximum Likelihood Detection**



**Quadratic Unconstrained Binary Optimization**



**Quantum Processing Unit**



**D-Wave 2000Q  
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**1. QUAMAX: SYSTEM DESIGN**

**2. QUANTUM ANNEALING & EVALUATION**

# QuAMax's Performance Metrics

- **One run on QuAMax includes multiple QA cycles.**  
Number of anneals ( $N_a$ ) is another input.
- **Solution (state) that has the lowest energy is selected as a final answer.**

**Evaluation Metric: How Many Anneals Are Required?**



**Target**

Bit Error Rate (BER)

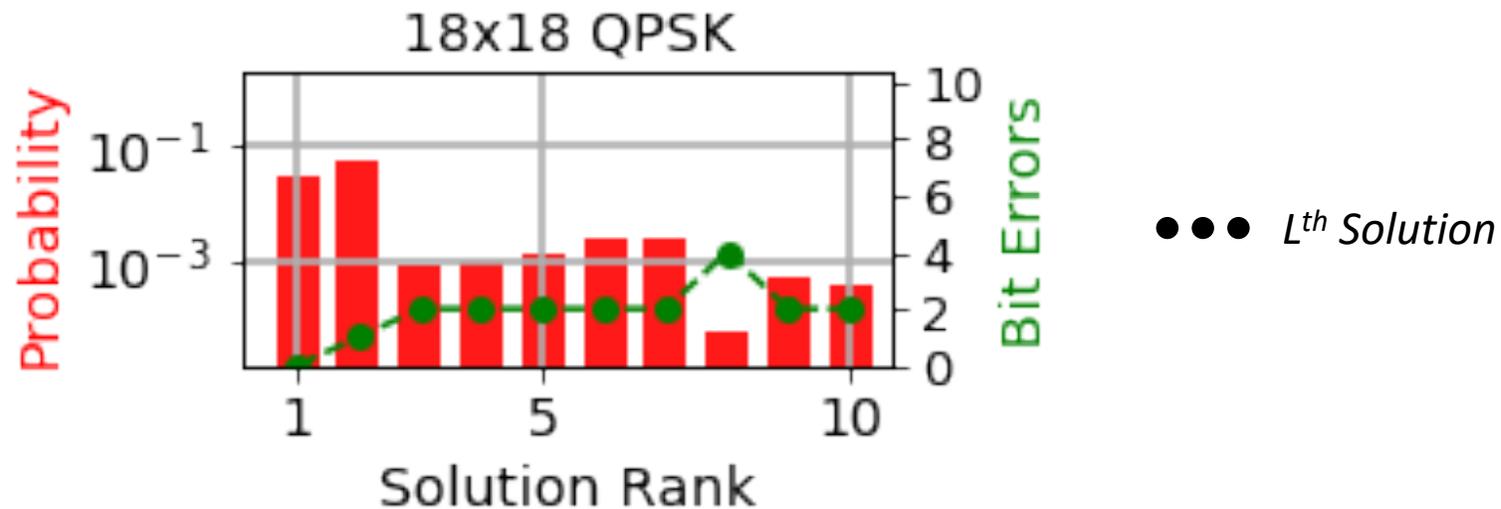


**Solution's Probability**

Empirical QA Results

# Experimental Methodology

1. Run enough number of anneals  $N_a$  for statistical significance.
2. Sort the  $L$  ( $\leq N_a$ ) results in order of QUBO energy.
3. Obtain the corresponding **probabilities** and **numbers of bit errors**.



# Wireless Performance Metric: Bit Error Rate (BER)

QuAMax's BER = BER of the lowest energy state after  $N_a$  Anneals

$$\mathbf{E}(BER(N_a)) = \sum_{k=1}^L \text{Probability of } k\text{-th solution being selected after } N_a \text{ anneals} \times \text{Corresponding BER of } k\text{-th solution}$$

||

Probability of [ never finding a solution better than k-th solution  
finding k-th solution at least once

This probability depends on number of anneals  $N_a$

**Expected Bit Error Rate (BER) as a Function of Number of Anneals ( $N_a$ )**

# QuAMax's Comparison Schemes

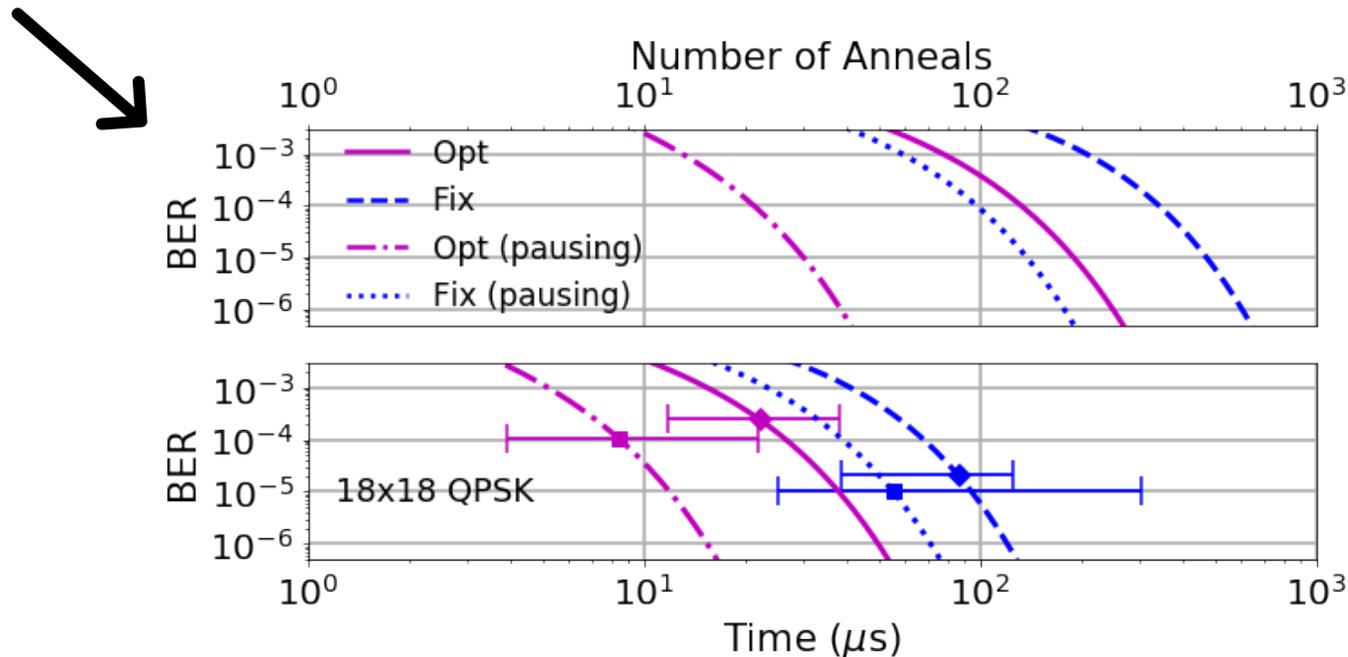
**QA parameters:** embedding, anneal time, pause duration, pause location, ...

- **Opt:** run with optimized QA parameters per instance (Oracle)
- **Fix:** run with fixed QA parameters per classification (QuAMax)

# Quantum Compute-Wireless Performance Metric: *Time-to-BER*

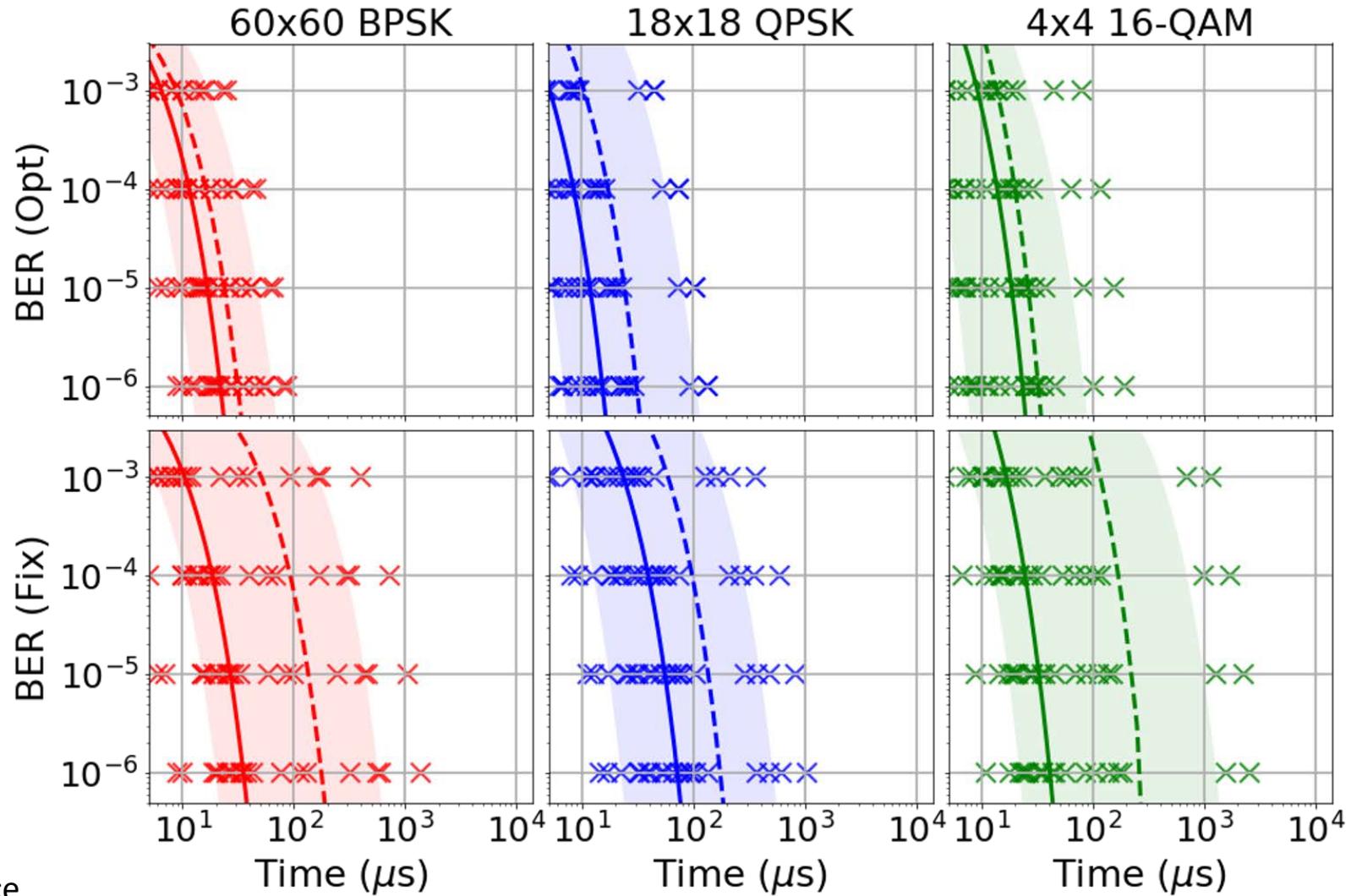
- **Opt**: run with optimized QA parameters per instance (**oracle**)
- **Fix**: run with fixed QA parameters per classification (**QuAMax**)

## Expected Bit Error Rate (BER) as a Function of Number of Anneals ( $N_a$ )

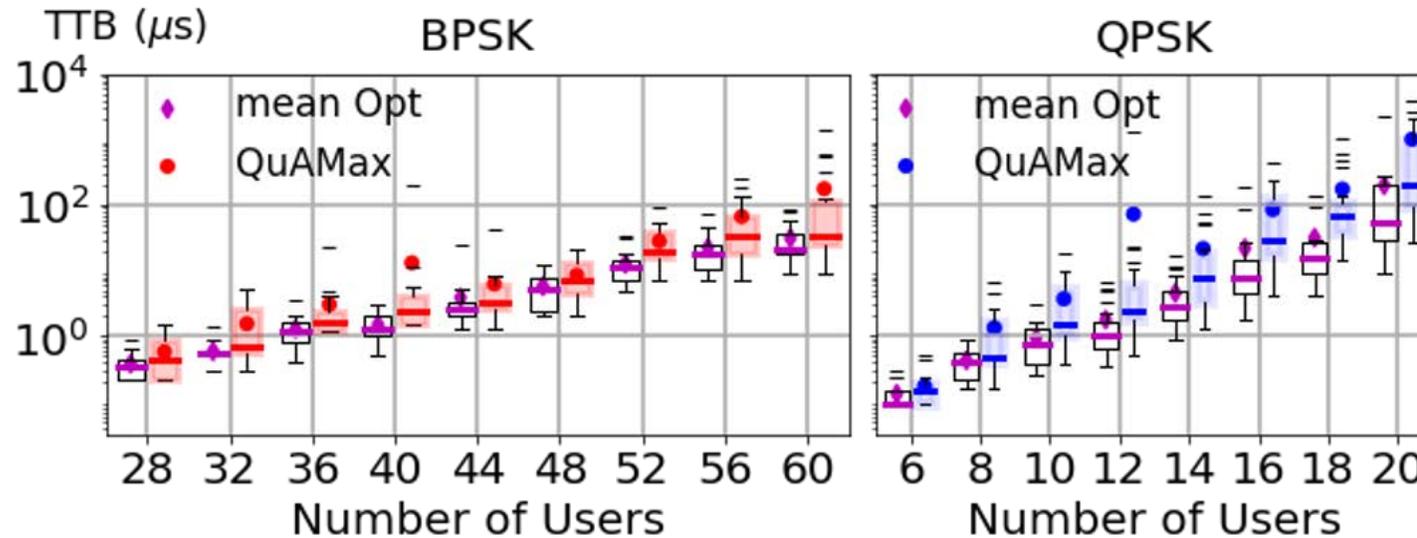


***Time-to-BER (TTB)***

# Time-to-BER for Various Modulations



# QuAMax's Time-to-BER ( $10^{-6}$ ) Performance

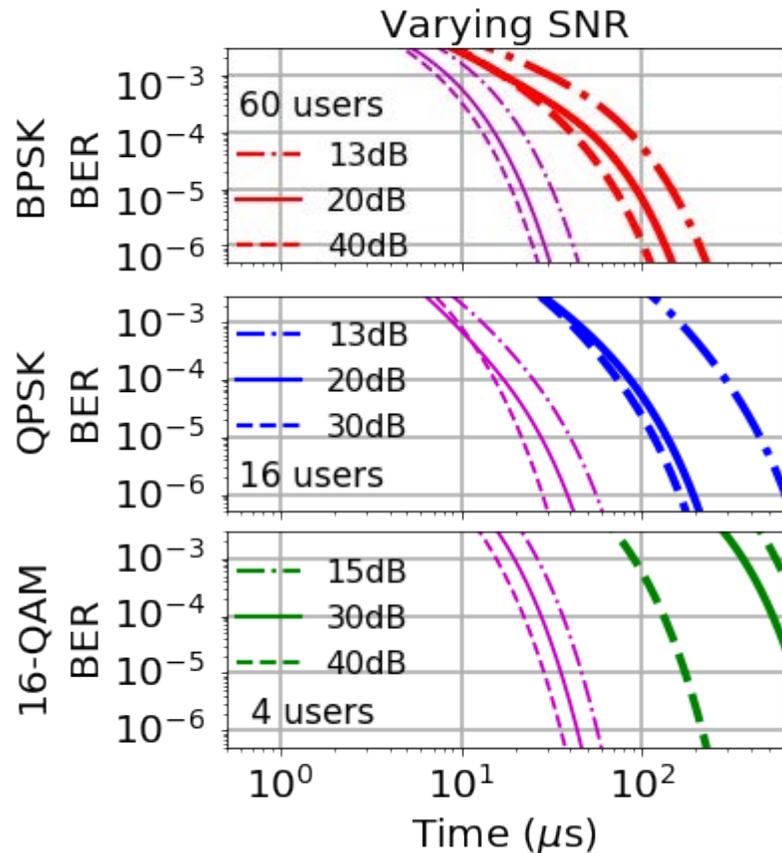


Practicality of  
Sphere Decoding

BPSK	QPSK	16-QAM	Complexity (Visited Nodes)
12 × 12	7 × 7	4 × 4	≈ 40 (♥)
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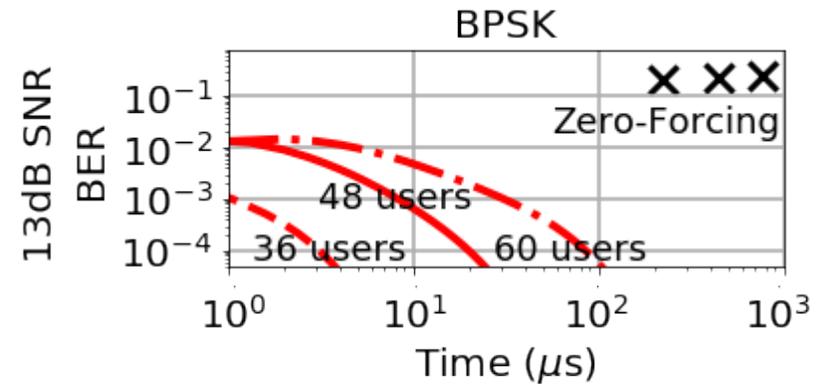
Well Beyond the Borderline of Conventional Computer

# QuAMax's Time-to-BER Performance with Noise



Same User Number  
Different SNR

- When user number is fixed, higher TTB is required for lower SNRs.

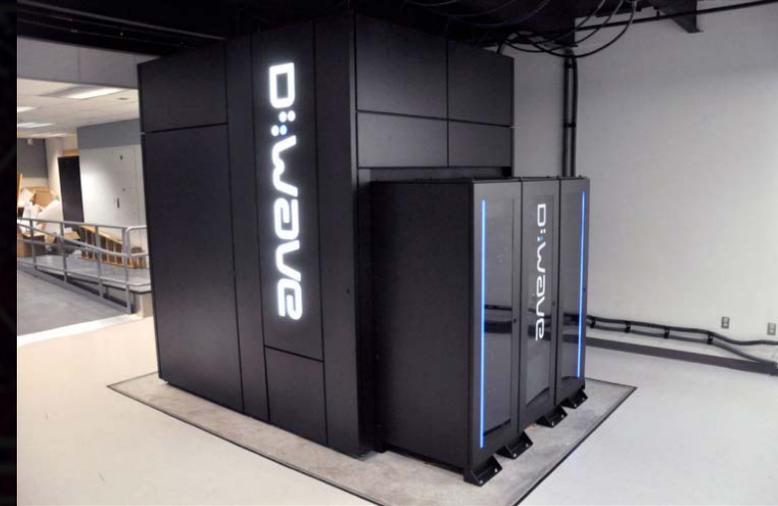


Comparison against Zero-Forcing

- Better BER performance than zero-forcing can be achieved.

## Practical Considerations

- Significant Operation Cost:  
About USD \$17,000 per year
- Processing Overheads (as of 2019):  
Preprocessing, Read-out Time,  
Programming Time = hundreds of *ms*



D-Wave 2000Q (hosted at NASA Ames)

## Future Trends in QA Technology

More Qubits (x2), More Flexibility (x2), Low Noise (x25),  
Advanced Annealing Schedule, ...

# Contributions

- **First application** of QA to MIMO detection
- **New metrics:** BER across anneals & Time-to-BER (TTB)
- **New techniques of QA:** Anneal Pause & Improved Range
- Comprehensive **baseline performance** for various scenarios

# Conclusion

- QA may hold the **potential** to overcome computational limits in **wireless networks**, but technology & integration yet to mature
- Our work **paves the way** for **quantum hardware and software** to contribute to improved performance envelope of **Massive MIMO**

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